References

C.K. Aidun, Y. Lu and E. Ding. Direct analysis of particulate suspensions with inertia using the discrete Boltzmann equation, *J. Fluid Mech.* **373**, 287, (1998).

T.B. Anderson and R. Jackson. A fluid mechanical description of fluidized beds. Equations of motion, *I&EC Fundamentals*, **6**, 527 (1967).

M.J. Andrews and P.J. O'Rourke. The multiphase particle-in-cell (MP-PIC) method for dense particulate flows. *Int. J. Multiphase Flow*, **22**, 379-402, (1996).

G.S. Arnold, D.A. Drew and R.T. Lahey. Derivation of constitutive equations for interfacial force and Reynolds stress for a suspension of spheres using ensemble averaging. *Chem Eng. Comm*. **86,** 43 (1989).

E.S. Asmolov. The inertial lift on a spherical particle in a plane Poiseuille flow at large channel Reynolds number, *J. Fluid Mech.* **381**, 63-87 (1999).

E.S. Asmolov. Dynamics of s spherical particle in a laminar boundary layer, *Fluid Dyn*. **25**, 886-890 (1990).

T.R. Auton. The lift force on a spherical body in a rotational flow, *J. Fluid Mech.* **183**, 199-218 (1987).

R. A. Bagnold. Fluid forces on a body in shear-flow; experimental use of 'stationary flow'. *Proc. R. Soc. Lond. A.* **340**, 147-171 (1974).

G.J. Barenblatt. Scaling, self-similarity, and intermediate asymptotics. *Cambridge Texts in Applied Math*. Cambridge University Press, UK (1996).

E. Barnea and J. Mizrachi. A generalized approach to the fluid dynamics of particulate systems, *Chem. Eng. Sci*, **5,** 171-189 (1973).

G.K. Batchelor. A new theory of the instability of a uniform fluidized bed, *J. Fluid Mech.* **193**, 75-110 (1988).

T. B. Benjamin. Note on formulas for the drag of a sphere, *J. Fluid Mech.*, **246**, 335 (1993).

J.F. Brady. *Stokesian Dynamics Simulation of Particulate Flows in Particulate Two-Phase Flow*, (M. Roco ed.) 971-998, Butterworth-Heinemann (1993).

H. Brenner. Hydrodynamic resistance of particles at small Reynolds numbers. *Adv. Chem. Engng*, **6**, 287, (1966).

F.P. Bretherton. Slow viscous motion round a cylinder in simple shear, *J. Fluid Mech*. **12**, 591, (1962).

M.O. Bristeau, R. Glowinski, J. Periaux. Numerical methods for the Navier-Stokes equations. Applications to the simulation of compressible and incompressible viscous flow, *Computer Physics Reports*, **6**, 73-187 (1987).

S. Chen and G.D. Doolen. Lattice Boltzmann method for fluid flows, *Annual Rev. Fluid Mech.*, **30**, 329-364 (1998).

P. Cherukat and J. McLaughlin. The inertial lift on a rigid sphere in a linear shear flow field near a flat wall, *J. Fluid Mech.* **263**, 1–18 (1994).

P. Cherukat, J. B. McLaughlin, and A. L. Graham. The inertial lift on a rigid sphere translating in a linear shear flow, *Int. J. Multiphase Flow*, **20**, 339--353 (1994).

H. Choi, Splitting method for the combined formulation of the fluid-particle problem, *Computer Methods in Applied Mechanics and Engineering*, **190**(11-12), 1367-1378 (2000).

H. Choi and D.D. Joseph. Fluidization by lift of 300 circular particles in plane Poiseuille flow by direct numerical simulation. *J. Fluid Mech.*, **438**, 101-128 (2001).

A.J. Chorin. A numerical method for solving incompressible viscous flow problems, *J. Comp. Phys.* **2**, 12–26 (1967).

A.J. Chorin. Numerical solution of the Navier-Stokes equations. *Math. Comput.* **22**, 745-762 (1968)a.

A.J. Chorin. On the convergence and approximation of discrete approximation to the Navier-Stokes equations, *Math. Comp.* **23**, 341–353 (1968)b.

A.J. Chorin. Numerical study of slightly viscous flow, *J. Fluid Mech.* **57**, 785–796 (1973).

R.G. Cox. The steady motion of a particle of arbitrary shape at small Reynolds numbers, *J. Fluid Mech.* **23,** 625-642 (1965).

R.G. Cox and H. Brenner. The lateral migration of solid particles in Poiseuille flow. Part 1 Theory. *Chem. Eng. Sci.* **23**, 147-163 (1968).

R.G. Cox and S.K. Hsu. The lateral migration of solid particles in a laminar flow near a plane. *Intl. J. Multiphase Flow*, **3**, 201 (1977)

R.G. Cox and S.G. Mason. Suspended particles in fluid flow through tubes, *Ann. Rev. Fluid Mech.* **3**, 291, (1971).

R.G. Cox, I.Y. Zia and S.G. Mason. Particle motions in sheared suspensions. XXV. Streamlines around cylinders and spheres, *J. Colloid Interface Sci.* **27**, 7 (1968).

C.T. Crowe, J.N. Chung, T.R. Troutt. Numerical models for two-phase turbulent flows. *Annual Review of Fluid Mechanics*, **28**, 11-43 (1996).

J.M. Dallavalle. *Micrometrics, 2nd Edition,* Pitman, London (1948)

J.M. Delhaye. General equations of two-phase systems and their applications to air-water bubble flow and to steam-water flashing flow. *ASME-AIChE Heat Transfer Conference,* Minneapolis (1969).

D.A. Drew. Averaged field equations for two-phase media, *Stud. Appl. Math.* **50**, 133 (1971).

D.A. Drew. Mathematical modeling of two-phase flow. *Ann. Rev. Fluid Mech.* **15**, 261 (1983)

D.A Drew and Passman. *Theory of Multicomponent Fluids*, Springer-Verlag, New York (1999).

D.A. Drew and L.A. Segel. Averaged equations for two-phase flows. *Stud. Appl. Math.* **50**, 205 (1971)

D.A. Drew and R.T. Lahey. Analytical modeling of multiphase flow, in *Particulate Two-Phase Flow*, ed. M.C. Roco, p. 510, Butterworth-Heinemann (1993)

R. Eichhorn and S. Small. Experiments on the lift and drag of spheres suspended in a Poiseuille flow, *J. Fluid Mech.* **20**, 513-527 (1964).

J. Feng, H.H. Hu, and D.D. Joseph. Direct simulation of initial values problems for the motion of solid bodies in a Newtonian fluid. Part 2: Couette and Poiseuille flows. *J. Fluid Mech.* **277**, 271-301 (1994).

J. Feng, P.Y. Huang, and D.D. Joseph. Dynamic simulation of the motion of capsules in pipelines. *J. Fluid Mech.* **286**, 201-207 (1995).

J. Feng, P.Y. Huang, and D.D. Joseph. Dynamic simulation of the sedimentation of solid particles in an Oldroyd B fluid, *J. Non-Newt. Fluid Mech.* **63**, 63–88 (1996).

J. Feng, D.D. Joseph, and P.Y. Huang. The motion and interaction of solid particles in viscoelastic liquids, *Rheol. Fl. Mech. Nonlin. Mat.* **217**, 123–133 (1996).

F. Feuillebois. Some theoretical results for the motion of solid shperical particles in a viscous fluid, in *Multiphase Science and Technology* (ed. G.F. Hewitt *et al*.), **4**, 583 Hemisphere, (1989).

A. Fortes, D.D. Joseph, and T.S. Lundgren. Nonlinear mechanics of fluidization of beds of spherical particles, *J. Fluid Mech.* **177**, 467–483 (1987).

P.V. Foscolo and L.G. Gibilaro. A fully predictive criterion for transition between particulate and aggregate fluidization, *Chem. Engng. Sci.* **39**, 1667-1675 (1984).

P.V. Foscolo and L.G. Gibilaro. Fluid dynamic stability of fluidized suspensions. The particle bed model. *Chem. Engng. Sci.* **42,** 1489-1500 (1987).

P.V. Foscolo, L.G. Gibilaro and S.P. Waldram. A Unified model for particulate expansion of fluidized bed and flow in porous media, *Chem Eng. Sci*. **38**, pp 125-1260 (1983).

A.W. Francis. Wall effect in falling ball method for viscosity, *Physics* **4**, 403-406 (1933).

R.C. Givler. An interpretation of solid phase pressure in slow fluid particle flows, *Int. J Multiphase Flow*, **13,** 717-722 (1977).

R. Glowinski. *Numerical Methods for Nonlinear Variational Problems*, Springer-Verlag, New York (1984).

R. Glowinski, T. Hesla, D.D. Joseph, T.-W. Pan, J. Periaux. Distributed Lagrange multiplier methods for particulate flows, in *Computational Science for the 21st Century*, M.O. Bristeau, G. Etgen, W. Fitzgibbon, J.L. Lions, J. Periaux, M.F. Wheeler eds., J. Wiley, Chichester, U.K., 270-279 (1997).

R. Glowinski, T.-W. Pan, T. I. Hesla, and D.D. Joseph. A distributed Lagrange multiplier/fictitious domain method for particulate flows, *Int. J. Multiphase Flow* **25**, 755-794 (1999).

R. Glowinski, T.-W. Pan, T.I. Hesla, D.D. Joseph, and J. Périaux. A distributed Lagrange multiplier/fictitious domain method for flows around moving rigid bodies: Application to particulate flow, *Int. J. Numer. Methods Fluids*, **30,** 1943-1066 (1999).

R. Glowinski, T.-W. Pan, T.I. Hesla, D.D. Joseph, and J. Périaux. A fictitious domain method with distributed Lagrange multipliers for numerical simulation of particulate flows, in *Domain Decomposition Method*s, (J. Mandel, C. Farhat, X.C. Cai eds.), American Mathematical Society, Providence, R.I., **10**, 121-137, (1998).

R. Glowinski, T.-W. Pan, T. I. Hesla, D. D. Joseph, J. Periaux. A distributed Lagrange multiplier/fictitious domain method for the simulation of flows around moving rigid bodies: Application to particulate flow, *Comp. Meth. Appl. Mech. Eng*., **184**, 241-268 (2000).

R. Glowinski, T.W. Pan, T.I. Hesla, D.D. Joseph, and J. Periaux. A fictitious domain method approach to the direct numerical simulation of incompressible viscous flow past moving rigid bodies: Application to particulate flow, *Journal of Computational Physics*, **169**, 363-426 (2001).

A.J. Goldman, R.G. Cox and H. Brenner. Slow viscous motion sphere parallel to a plane wall—Couette Flow, *Chem. Eng. Sci.* **22**, (1967).

R. A. Gore and C. T. Crowe. Effect of particle size on modulating turbulent intensity, *Int. J. Multiphase Flow* **16**, 279 (1989).

P. Hansbo. The characteristic streamline diffusion method for the time-dependent incompressible Navier-Stokes equations, *Comp. Meth. Appl. Mech. Eng.* **99**, 171–186 (1992).

T. I. Hesla. The dynamical simulation of two-dimensional fluid/particle systems, unpublished notes (1991).

E.J. Hinch. An averaged equation approach to particle interactions in a fluid suspension, *J. Fluid Mech*. **83**, 695 (1977).

T. I. Hesla. Collisions in viscous fluids, in preparation (1998).

B.P. Ho and L.G. Leal. Inertial migration of rigid spheres in two-dimensional unidirectional flows, *J. Fluid Mech*. **65**, 365 (1974).

K. Hofler, M. Muller, S. Schwarzer and B. Wachmann. Interacting particle-liquid systems, in *High Performance Computing in Science and Engineering '98*, E. Krause and W. Jager eds., 54-64, Springer-Verlag, Berlin (1999).

S. Hou, Q. Zou, S. Chen, G. Doolen and A. Cogley. Simulation of cavity flow by the Lattice Boltzmann method, *J. Comp. Physics,*. **118**, 329-347, (1995)

H. H. Hu. Direct simulation of flows of solid-liquid mixtures, *Int. J. Multiphase Flow* **22**, 335–352 (1996)a.

H. H. Hu. Numerical simulation of channel Poiseuille flow of solid-liquid mixtures, in *Proceedings of the Fluids Engineering Division Summer Meeting*, American Society of Mechanical Engineers, **236**, 97–103 (1996)b.

H.H. Hu. Direct numerical simulations of fluid solid systems. See *http://www.lrsn.upenn.edu/~howard/homepage.html* (to appear 2001).

H.H. Hu and D.D. Joseph. Lift on a sphere near a plane wall in a second-order fluid. *J. Non-Newtonian Fluid Mech.* **88**, 173-184 (1999).

H.H. Hu, N.A. Patankar and M.Y. Zhu, Direct numerical simulation of fluid-solid systems using the arbitrary Lagrangian-Eulerian technique, *J. of Computational Physics*, **169** (427-462). (2001).

H.H. Hu. D.D. Joseph, and M. J. Crochet. Direct simulation of fluid particle motions, *Theor. Comp. Fluid Dyn.* **3**, 285 (1992)a.

H.H. Hu. D.D. Joseph, and A. Fortes. Experiments and direct simulations of fluid particle motion, *Int. Vid. J. Eng. Res.* **2**, 17 (1992)b.

H.H. Hu, N.A. Patankar and M.-Y. Zhu. Direct numerical simulations of fluid-solid systems using the Arbitrary-Lagrangian-Eulerian technique, *J. of Computational Physics,* **169**, 427-462. (2000).

P.Y. Huang, J. Feng, H. H. Hu, and D.D. Joseph. Direct simulation of the motion of solid particles in Couette and Poiseuille flows of Viscoelastic Fluids, *J. Fluid Mech.* **343**, 73–94 (1997).

P.Y. Huang, J. Feng, and D.D. Joseph. The turning couples on an elliptic particle settling in a vertical channel, *J. Fluid Mech.* **271**, 1–16 (1994).

P.Y. Huang, H. H. Hu, and D.D. Joseph. Direct simulation of the sedimentation of elliptic particles in Oldroyd B fluids, *J. Fluid Mech.* 362, 297–326 (1998).

P.Y. Huang and D.D. Joseph. Effects of shear thinning on migration of neutrally buoyant particles in pressure driven flow of Newtonian and viscoelastic fluid, *J. Non-Newtonian Fluid Mech.*, **90**, 159-185 (2000).

N.A. Patankar, P.Y. Huang, T. Ko and D. D. Joseph. Lift-off of a single particle in Newtonian and viscoelastic fluids by direct numerical simulation, *J. Fluid Mech.,* **438**, 67-100, 2001.

A. Huerta and W. K. Liu. Viscous flow with large free surface motion, *Comp. Meth. Appl. Mech. Eng.* **69**, 227–324 (1988).

G. Iooss and D.D. Joseph. *Elementary Stability and Bifurcation Theory*. Springer, 2nd edition (1990).

R. Jackson. Locally averaged equations of motion for a mixture of identical spherical particles and a Newtonian fluid. *Chem. Eng. Sci.* **52**, 2457 (1997)

A. Johnson and T. E. Tezduyar. Simulation of multiple spheres falling in a liquid-filled tube, *Comp. Meth. Appl. Mech. Eng.* **134**, 351–373 (1996).

A. Johnson and T.E. Tezduyar. Advanced mesh generation and update methods for 3D flow simulations, *Computational Mech.* **23**, 130-143 (1999).

D.D. Joseph. *Fluid Dynamics of Viscoelastic Liquids*, Springer-Verlag, New York (1990)a.

D.D. Joseph. Generalization of the Foscolo-Gibilaro analysis of dynamic waves, *Chem. Eng. Sci.* **45**(2), 411-414 (1990)b.

D.D. Joseph. Finite size effects in fluidized suspension experiments, in *Particulate Two-Phase Flow*, (M. C. Roco, ed.), pp. 300–324. Butterworth-Heinemann (1993).

D.D. Joseph. Flow induced microstructure in Newtonian and viscoelastic fluids, in *Proceedings of the Fifth World Congress of Chemical Engineering, Particle Technology Track* **6**, 3–16, American Institute of Chemical Engineers, San Diego Keynote presentation (Paper no. 95a, Second Particle Technology Forum). San Diego, California (1996).

D.D. Joseph. Flow induced microstructure in Newtonian and viscoelastic fluids. In *Proc. 5th World Congress of Chem. Engng, Particle Technology Track, San Diego, July 14-18. AIChE*, **6**, 3-16 (1996).

D.D. Joseph. Power law correlations for lift from direct numerical simulation of solid-liquid flow, *Int. J. Multiphase Flow*, accepted (2001).

D.D. Joseph, J. Belanger and G.S. Beavers. Breakup of a liquid drop suddenly exposed to a high speed airstream. *Int. J. Multiphase Flow*, **25**, 1263-1303 (1999).

D.D. Joseph and J. Feng. A note on the forces that move particles in a second order fluid, *J. Non-Newt. Fluid Mech.* **64**, 299–302 (1996).

D.D. Joseph, A. Fortes, T.S. Lundgren, and P. Singh. Nonlinear mechanics of fluidization of beds of spheres, cylinders and disks in water, in *Advances in Multiphase Flow and Related Problem*s, (G. Papanicolau, ed.), pp. 101–122. SIAM (1987).

D.D. Joseph and T. Liao. Potential flows of viscous and viscoelastic liquids, *J. Fluid Mech.,* **265**, 1-23, (1994).

D.D. Joseph and T.S. Lundgren. Ensemble averaged and mixture theory equations for incompressible fluid-particle suspensions, *Int. J. Multiphase Flow* **16**, 35 (1990).

D.D. Joseph, Y. J. Liu, M. Poletto, and J. Feng. Aggregation and dispersion of spheres falling in viscoelastic liquids, J. Non-Newt. Fluid Mech. 54, 45–86 (1994).

D.D. Joseph and D. Ocando, 2002. Slip velocity and lift, **454**, *J. Fluid Mech*., 263-286.

D.D. Joseph and J.C. Saut. Short-wave instabilities and ill-posed initial-value problems, *Theoretical Comput. Fluid Dynamics*, **1,** 191-227 (1990).

A. Karnis and S.G. Mason. Particle motions in sheared suspensions. XIX: Viscoelastic media, *Trans. Soc. Rheol.* **10**, 571–592 (1966).

T. Kataoka, T. Kitano, M. Sasahara and K. Nishijima. Viscosity of particle filled polymer melts, *Rheol. Acta*, **17,** 149-155 (1978).

L.R. Kern, T.K. Perkins, and R.E. Wyant. The mechanics of sand movement in fracturing, *Petroleum Transactions, AIME* **216**, 403–405 (1959).

S. Kim and S.J. Karrila. *Microhydrodynamics: Principles and Selected Applications*, Butterworth-Heinemann, Boston (1991).

M. R. King and D. T. Leighton, Jr. Measurement of the inertial lift on a moving sphere in contact with a plane wall in shear flow, *Phys. Fluids*, **9**, 1248--1255 (1997).

M.G. Knepley, V. Sarin, and A.H. Sameh. Parallel simulation of particulate flows, *in Proceedings of the Fifth International Symposium on Solving Irregularly Structured Problems in Parallel: IRREGULAR '98*, Lecture Notes in Computer Science, Springer-Verlag, No. **1457**, pp. 226-237, Aug. 1998.

G.P. Krishnan and D. T. Leighton. Inertial lift on a moving sphere in contact with a plane in a shear flow, *Phys Fluids* **7**(11), 2538–2545 (1995).

A. Kuethe and C.Y. Chow. *Foundations of Aerodynamics, 5th Ed*. John Wiley & Sons (1998).

A.J.C. Ladd. Numerical simulations of particulate suspensions via a discretized Boltzmann equation, Part 1, Theoretical foundation, *J. Fluid Mech.* **271**, 285 (1994a).

A.J.C. Ladd. Numerical simulations of particulate suspensions via a discretized Boltzmann equation, Part 2, Numerical results, *J. Fluid Mech.* **271**, 311 (1994b).

A.J.C. Ladd. Sedimentation of homogeneous suspensions of non-Brownian spheres, *Phys. Fluids.* **9**, 491 (1997).

L.G. Leal. Particle motions in a viscous fluid, *Ann. Rev. Fluid Mech.* **12**, 435 (1980).

D.T. Leighton and A. Acrivos. The lift on a small sphere touching a plane in the presence of a simple shear flow, *Z. Angew. Math. Phys* **36**, 174–178 (1985).

E.W. Lewis, and E.W. Bowerman. Fluidization of solid particles in liquids, *Chem. Engng. Progr*., **48**, 603 (1952).

W.K. Lewis, E.R. Gilliand and W.C. Bauer. *Indust. Engrg. Chem.*, **41**, 1104 (1949).

C.-J. Lin, J.H. Peery and W.R. Schowalter. Simple shear flow round a rigid sphere: inertial effects and suspension rheology, *J. Fluid Mech.*, **44**, part 1, 1-17 (1970).

Y.J. Liu and D.D. Joseph. Sedimentation of particles in polymer solutions. *J. Fluid Mech.* **255**, 565-595 (1993).

G.I. Marchuk. Splitting and alternating direction methods, in *Handbook of Numerical Analysis, Vol. I,* P.G. Ciarlet and J.L. Lions eds., North-Holland, Amsterdam, 197-462 (1990).

B.A. Maury and R. Glowinski. Fluid-particle flow: a symmetric formulation, *C. R. Acad. Sci. Paris T. S´erie I*, **324,** 1079–1084 (1997).

M.R. Maxey, and B.K. Patel. Force-coupled simulations of particle suspensions at zero and finite Reynolds numbers, *ASME FEDSM*, 97-3188 (1997).

M.R. Maxey, B.K. Patel, E.J. Chang, L.P. Wang. Simulations of dispersed turbulent multiphase flow, *Fluid Dynamics Res.*, **20**, 143-156 (1997).

M.R. Maxey and J.J. Riley. Equation of motion for a small rigid sphere in a nonuniform flow, *Phys. Fluids A* **26**, 883 (1983).

J.B. McLaughlin. Inertial migration of a small sphere in linear shear flows, *J. Fluid Mech*., **224**, 261-274 (1991).

J.B. McLaughlin. Numerical computation of particle-turbulent interaction, *Int. J. Multiphase Flow*, **20**, 211-132 (1994).

A.B. Metzner. Rheology of suspensions in polymeric liquids, *J. Rheol.* **29,** 739-776, (1985).

A.M. Mollinger and F.T.M. Nieuwstadt. Measurement of the lift force on a particle fixed to the wall in the viscous sublayer of a fully developed boundary layer, *J. Fluid Mech.* 316, 285-306 (1996).

R.I. Nigmatulin. Spatial averaging in the mechanics of heterogeneous and dispersed systems, *Int. J. Multiphase Flow* **5,** 353 (1979).

K. G. Nolte. Fluid flow considerations in hydraulic fracturing, *SPE* 18537 (1988)a. See also Errata submitted July 1992 for SPE 18537, available from SPE Book Order Department.

K. G. Nolte. Principles for fracture design based on pressure analysis, in *SPEPE*, 22–30 (Feb 1998)b.

T. Nomura and T. J. R. Hughes. An arbitrary Lagrangian-Eulerian finite element method for interaction of fluid and a rigid body, *Comp. Meth. Appl. Mech. Eng.* **95**, 115–138 (1992).

J. Nunziato, S. Passman, R.C. Givler, D. MacTigire and J Brady. Continuum theories for suspensions, *Advancements in Aerodynamics, Fluid Mechanics and Hydraulics (Proc ASCE S.N.)* 465-472 (1986).

M. Oettli and A. Sameh. *A Partitioning Scheme for the Parallel Solution of Banded Linear Systems,* Technical Report, Computer Science Dept., Univ. of Minnesota, Minneapolis (1997).

E. Ory, H.N. Oguz and A. Prosperetti. Physalis: a new *O*(*N*) method for the numerical simulation of particle flows, *Proceedings of ASME FEDSM'00,* ASME 2000 Fluids Engineering Divisions Summer Meeting, June 11-15, Boston, MA (2000).

T.-W. Pan, D.D. Joseph and R. Glowinski. Modeling Rayleigh-Taylor instability of a sedimenting suspension of several thousand circular particles in direct numerical simulation. *J. Fluid Mech*. **434**, 23-37 (2001).

T.-W. Pan, V. Sarin, R. Glowinski, A. H. Sameh, and J. Périaux. A fictitious domain method with distributed Lagrange multipliers for the numerical simulation of particulate flow and its parallel implementation, in *Proceedings of the 10th Parallel CFD Conference* (1998), and in *Parallel Computational Fluid Dynamics, Development and Applications of Parallel Technology,* C.A. Lin, A. Ecer, N. Satofuka, P. Fox, and J. Periaux eds. North-Holland, Amsterdam, 467-474 (1999).

T.W. Pan, D.D. Joseph, R. Bai, R. Glowinski, V. Sarin. Fluidization of 1204 spheres: simulation and experiment. *J. Fluid Mech.*, **451**, 169-191 (2002)

T.-W. Pan. Numerical simulation of the motion of a ball falling in an incompressible viscous fluid. *Compte Rendus Acad. Sci. Paris,* **327**, 1035-1038 (1999)b.

Y. Pan and S. Banerjee. Numerical investigation of the effects of large particles on wall-turbulence, *Phys Fluids,* **9**(12), 3786 – 3806 (1997).

N.A. Patankar. *Numerical simulation of particulate two-phase flow.* Ph.D. thesis. University of Pennsylvania (1997).

N.A. Patankar, Y. Huang, T. Ko, and D.D. Joseph. Lift-off of a single particle in Newtonian and viscoelastic fluids by direct numerical simulation. *J. Fluid Mech.,* **438**, 67-100 (2001).

N.A. Patankar and D.D. Joseph. Lagrangian numerical simulation of particulate flows. *Int. J. Multiphase Flow,* **27**(10), 1685-1706 (2001).

N.A. Patankar and D.D. Joseph. Modeling and numerical simulation of particulate flows by the Eulerian-Lagranian approach, *Int. J. Multiphase Flow,* **27**(10), 1659-1684 (2001).

N.A. Patankar, D.D. Joseph, J. Wang, R.D. Barree, M. Conway and M. Asadi. Power law correlations for sediment transport in pressure driven channel flows, *Int. J. Multiphase Flow*, to appear (2002).

N.A. Patankar, T. Ko, H.G. Choi and D.D. Joseph. A correlation for the lift-off of many particles in plane Poiseuille flows of Newtonian fluids, *J. Fluid Mech*., **445**, 55-76 (2001).

N.A. Patankar, P. Singh, D.D. Joseph, R. Glowinski and T.-W. Pan. A new formulation of the distributed Lagrange multiplier/fictitious domain method for particulate flows, *Int. J. Multiphase Flow,* **26**(9), 1509-1524 (2000).

C. S. Peskin. Numerical analysis of blood flow in the heart, *J. Comp. Phys*. **25**, 220–252 (1977).

C. S. Peskin. Lectures on mathematical aspects of physiology, in *Lectures in Applied Mathematics. Volume 19: Mathematical Aspects of Physiology*, (Frank C. Hoppensteadt, ed.), pp. 69–107. American Mathematical Society, Providence, Rhode Island (1981).

C.S. Peskin and D.M. McQueen. Modeling prosthetic heart valves for numerical analysis of blood flow in the heart, *J. Comp. Phys.* **37**, 113–132 (1980).

M. Poletto and D.D. Joseph. Effective density and viscosity of a suspension, *J. Rheol*., **39**(2), 323-343 (1995).

I. Proudman and J.R.A. Pearson. Expansions at small Reynolds numbers for the flow past a sphere and a circular cylinder, *J. Fluid Mech.* **2**, 237-262 (1957).

J. F. Richardson and W. N. Zaki. Sedimentation and fluidization: Part I, *Trans. Instn. Chem. Engrs.* **32**, 35–53 (1954).

P.N. Rowe. A convenient empirical equation for estimation of the Richardson-Zaki exponent, *Chem. Engng. Sci.* **42**, 2795-2796 (1987).

S. I. Rubinow and J.B. Keller. The transverse force on a spinning sphere moving in a viscous fluid. *J. Fluid Mech.* **11**, 447-459 (1961).

D. Qi. Lattice-Boltzmann simulations of particles in non-zero-Reynolds-number flows, *J. of Fluid Mech.* **385,** 41 (25 April 1999)

Y. Qian, D. d'Humieres and P. Lallemand. Lattice BGK models for Navier-Stokes equation, *Europhys. Lett.* **17**, 479 (1992).

M. J. Riddle, C. Narvaez, and R. B. Bird. Interactions between two spheres falling along their line of centers in a viscoelastic liquid, *J. Non-Newt. Fluid Mech.* **2**, 23–35 (1977).

M.C. Roco. One-equation turbulence modeling of incompressible mixtures. *Encyclopedia of Fluid Mechanics,* Vol 10, chapter 1, 1-68, Gulf. Pub. Co, (1990).

M.C Roco and N. Balakrishnam. Multi-dimensional flow analysis of solid-liquid mixtures. *J. Rheology*, **29** (4), 431-456 (1985).

S.I. Rubinow and J.B. Keller. The transverse force on a spinning sphere moving in a viscous fluid. *J. Fluid Mech.*, **11**, 447 (1961).

Y. Saad and M. Sosonkina. Distributed Schur complement techniques for general sparse linear systems, *UMSI* 97/159, University of Minnesota Supercomputer Institute (1997).

P.G. Saffman. The lift on a small sphere in a slow shear flow, *J. Fluid Mech.* **22**, 385 (1965); and Corrigendum *J. Fluid Mech*. 31, 624 (1968).

P.G. Saffman. On the motion of small spheroidal particles in a viscous liquid, *J. Fluid Mech.* **1,** pp. 540-543 (1956).

A.H. Sameh and V. Sarin. Hybrid parallel linear system solvers, in *Proceedings of the Fourth Japan-US Symposium on Finite Element Methods in Large-Scale Computational Fluid Dynamics* (1998).

P. Saramito. A new scheme algorithm and incompressible FEM for viscoelastic fluid flows, *Math. Modeling and Num. Anal.* **28**, 1–35 (1994).

V. Sarin. *Spectral Analysis of Balanced CG Methods*, Technical Report, Computer Science Dept., Univ. of Minnesota, Minneapolis (1993).

V. Sarin and A.H. Sameh. An efficient iterative method for the generalized Stokes problem, *SIAM J. Sci. Stat. Comp.* **19**(1), 206–226 (1998)a.

V. Sarin and A. H. Sameh. Multilevel algorithms for elliptic PDEs, in *Proceedings of the Fifth Copper Mountain Conference on Iterative Methods* (1998)b.

J.A Schonberg and E.J. Hinch. Inertial migration of a sphere in Poiseuille flow, *J. Fluid Mech*. **203**, 517 (1989).

G. Segré and A. Silberberg. Radial Poiseulle flow of suspensions, *Nature*, **189**, 209 (1961).

G. Segré and A. Silberberg. Behaviour of macroscopic rigid spheres in Poiseuille flow, Part I, *J. Fluid Mech.*, **14**, 115 (1962).

A. Shields. Anwendung der aenlichkeitsmechanik und der turbulenzforschung auf die geschiebebewegung. *Mitteilungen der Preussischen Versuchsanstalt fur Wasserbau und Schiffbau*, Berlin, Germany, translated to English by W.P. Ott and J.C. Uchelen, California Institute of Technology, California (1936).

D. Silvester and A. Wathen. Fast iterative solution of stabilized Stokes systems. Part II: Using general block preconditioners, *SIAM J. Num. Anal*. **31**(5), 1352–1367 (1994).

P. Singh, P. H. Caussignac, A. Fortes, D.D. Joseph, and T. S. Lundgren. Stability of periodic arrays of cylinders across the stream by direct simulation, *J. Fluid Mech.* **205**, 553–571 (1989).

P. Singh and D.D. Joseph Sedimentation of a sphere near a wall in an Oldroyd-B fluid. *J. Non-Newtonian Fluid Mech.*, **94**(2-3), 179-203 (2000).

P. Singh, D.D. Joseph, T. Hesla, R. Glowinski and T.-W. Pan. A distributed Lagrange multiplier/fictitious domain model for viscoelastic particulate flows, *J. Non-Newtonian Fluid Mech.*, **91**, 165-188 (2000).

P. Singh and L.G. Leal. Finite-element simulation of the start-up problem for a viscoelastic fluid in an eccentric cylinder geometry using a third order upwind scheme, *Theor. Comp. Fluid Dyn.* **5**, 107–137 (1993).

P. Singh and L.G. Leal. Computational studies of the FENE dumbbell model in a co-rotating two-roll mill, *J. Rheology* **38**(3), 485–517 (1994).

D.M. Snider, P.J. O'Rourke, and M.J. Andrews. Sediment flow in inclined vessels calculated using a multiphase particle-in-cell model for dense particle flow, *Int. J. Multiphase Flow* **24**, 1359-1382 (1998).

A. Tehrani, P.S. Hammond, and A.T. Unwin. *Experimental study of particle migration in suspensions undergoing Poiseuille flow*, presentation given at the Society of Rheology 66th Annual Meeting (1994).

D.G. Thomas. Transport characteristics of suspension: VIII. A note on the viscosity of Newtonian suspension of uniform spherical particles. *J. Colloid Sci.* **20**, 267 (1965).

S. Turek. A comparative study of time-stepping techniques for the incompressible Navier-Stokes equations: From fully implicit non-linear schemes to semi-implicit projection methods, *Int. J. Num. Meth. Fluids* **22**, 987–1011 (1996).

A.T. Unwin and P.S. Hammond. Computer simulations of proppant transport in a hydraulic fracture, *SPE* 29649, Society of Petroleum Engineers Presented at the Society of Petroleum Engineers Western Region Meeting (1995).

P. Vasseur and R.G. Cox. The lateral migration of a spherical particle in two-dimensional shear flows, *J. Fluid Mech.* **78**, 385 (1976).

G.B. Wallis. *One Dimensional Two-phase Flow*, McGraw-Hill, New York (1969).

G.B. Wallis. The averaged Bernoulli equation and macroscopic equations of motion for the potential flow of a two-phase dispersion, *Int. J. Multiphase Flow* **17**, 683 (1991).

J. Wang, D.D. Joseph and M. Conway. Carpet traction correlations for proppant transport, in preparation (2002).

S. Whitaker. Advances in the theory of fluid motion in porous media, *Ind. Eng. Chem.* **61**, 14 (1969).

R.H. Wilhelm, and M. Kwauk. Fluidization of solid particles, *Chem. Engng. Progr*. **44,** 201 (1948).

J. Ye, and M.C. Roco. Particle rotation in a Couette flow, *Phys. Fluids,* **4**(2), 220-224 (1992).

D.Z. Zhang and A. Prosperetti. Averaged equations for inviscid disperse two-phase flow, *J. Fluid Mech*. **267**, 185-219 (1994).

D.Z. Zhang and A. Prosperetti. Momentum and energy equations for disperse two-phase flows and their closure for dilute suspensions, *Int. J. Multiphase Flow* **23**, 425 (1997).

M.-Y. Zhu. *Direct numerical simulation of the solid-liquid flows of Newtonian and viscoelastic fluids.* Ph.D. thesis. University of Pennsylvania (2000).

List of symbols

English Symbols

$\mathbf{1}$	Identity tensor
A	Area of cross section
$\mathbf{A}$	Configuration tensor (= $\boldsymbol{\tau}_E + (\eta_E / \lambda_1)\mathbf{1}$)
a	Radius of the particles
$\mathbf{b}$	Body force
D	Diameter of cylindrical tube
$\boldsymbol{D}$	Drag force
d	Diameter of the particles
$\overline{d}$	Average diameter of the particles
$d_{i,j}$	Distance between the center of the i-th and j-th particles
$\mathbf{e}_x$	Unit vector in the x-direction
$\mathbf{e}_y$	Unit vector in the y-direction
$\mathbf{F}$	Force
F_i'	Short-range repulsive force exerted on the i-th particle (Eq. VII.9)
$F_{i,j}^{p}$	Particle-particle repulsive force (Eq. VII.10)
$F_{i,j}^{p}$	Particle-wall repulsive force (Eq. VII.11)
$\mathbf{g}$	Acceleration due to gravity
H	Indicator function
H	Height of the channel
H_e	Equilibrium height of the fluidized bed
h_v	Velocity mesh size
h_p	Pressure mesh size
$\boldsymbol{I}$	Moment of inertia tensor
L	Channel length
$\boldsymbol{L}$	Lift force
$\boldsymbol{L}'$	Lift force per unit length
$\boldsymbol{L}_e$	Elastic component of lift force
$\boldsymbol{L}_p$	Pressure component lift force
$\boldsymbol{L}_s$	Shear component lift force
M	Particle mass
N	Number of particles
$\mathbf{n}$	Unit outward normal vector on the particle surface
n_ℓ	Growth rate of the wave amplitude
p	Pressure
$\overline{p}$	Average pressure gradient
p_a	Atmospheric pressure

$\mathbf{r}$	Position vector of the point w.r.t. the center of mass of the particle
$\mathbf{T}$	Torque
$\mathbf{T}$	Dimensionless stress
$\mathbf{T}^*$	Stress tensor (Chap. 6)
$\mathbf{T}_f$	Stress for the fluid phase (Chap. 6)
t	Time
$\mathbf{t}$	Traction on the particle surface
$\mathbf{U}$	Translational velocity of the particles
U_m	Mean velocity of the fluid (Chap. 9)
U_s	Slip velocity (= $U_f - U_p$)
$\mathbf{u}$	Velocity of the fluid
$\mathbf{V}_c$	Composite velocity (= $\varepsilon\mathbf{V}_f + \phi\mathbf{V}_s$) (Chap. 6)
$\mathbf{V}_f$	Average velocity of the fluid phase (Chap. 6)
$\mathbf{V}_m$	Mass averaged velocity (Chap. 6)
V_p	Volume per unit length of the particles (Chaps. 9 and 11)
$\mathbf{V}_s$	Average velocity of the solid phase (Chap. 6)
W	Width of domain
$\mathbf{X}$	Coordinate of the center of mass of the particle

Greek Symbols

ε	Volume fraction of the fluid phase
ε_p and ε_w	Stiffness parameter for the collision scheme
Φ_e	Elastic lift fraction
Φ_p	Pressure lift fraction
Φ_s	Shear lift fraction
ϕ	Volume fraction of the solid phase
Γ	Circulation
γ	Surface tension
$\dot{\gamma}$	Shear rate
$\dot{\gamma}_w$	Shear rate at the wall
η	Viscosity of the fluid
η_2	Effective viscosity of the composite (Eq. VIII.8)
η_E	Elastic viscosity
η_m	Effective viscosity of the composite (Chap. 14)
$\boldsymbol{\lambda}$	Lagrange multiplier field
λ	Wavelength (Chap. 8)
λ_1	Relaxation time
λ_2	Retardation time
ν	Kinematic viscosity
$\boldsymbol{\Pi}$	Pressure in a periodic domain
π	3.141592
$\boldsymbol{\theta}$	Angular position of the particle

ρ_f	Density of the fluid
ρ_2	Effective density of the composite (Chap. 8)
$\boldsymbol{\sigma}$	Stress tensor
τ	Shear stress
$\boldsymbol{\tau}_E$	Elastic stress
υ	composite velocity
υ_m	Migration velocity (Chap. 10)
Ω	Computational domain, angular velocity
Ω_f	Fluid domain
Ω_s	Particle domain
Ω_f	Angular velocity of the fluid
Ω_p	Angular velocity of the particle
Ω_s	Slip angular velocity ($=\Omega_f - \Omega_p$)
ω	Angular velocity of the particle

Dimensionless Groups

C_D	Drag coefficient
De	Deborah number
E	Elasticity number De/R
G	Gravity parameter
R	Shear Reynolds number
R_G	Gravity Reynolds number
R_s	Slip Reynolds number

Subscript

2D	Two dimensional
3D	Three dimensional
0	Initial Value t = 0 (Chap. 14)
c	Composite
E	Elastic
f	Fluid
i and *j*	*i*-th and *j*-th particle
p	particle
s	solid
sed	Sedimentation
T	Total
∞	$t \rightarrow \infty$

Index

E

F

G

H

I

J

K

L

M

N

O

P

R

S

T

U

V

W